The partitioned solution of Ref. 2 uses smaller matrices than the $[Z^{-1}]$ technique of Ref. 1. However, it becomes necessary to choose in advance which of the $\alpha$ are to be surplus and which can safely be put in (1, 1) correspondence with the nodal deflections. This problem does not arise with the $[Z^{-1}]$ technique.

Arithmetically it is not yet clear which is the better. Methods of inversion are known which take full advantage of symmetry, using just over $\frac{1}{2}N^2$ words of storage and $\frac{1}{2}N^3$ multiplications to invert an $N \times N$ matrix. A modified Waugh and Dwyer technique achieves this, as does the row-inversion technique discussed in Ref. 3. These techniques are normally recommended only for positive definite matrices. It is the authors' feeling that such a technique is applicable with a modified order of pivoting. If such a technique is applicable, the partitioned solution will use only slightly fewer operations than the $[Z^{-1}]$ solution. Nothing is yet known concerning the relative numerical accuracy.

### References


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[3] Asplund, S. O., *Structural Mechanics* (Chalmers Tekniska Hogskola, Gothenburg, 1963), Vol. II, Sec. Mg.


## Comments on "Sputtering in the Upper Atmosphere"


R. V. STUART*

*Litton Industries, Minneapolis, Minn.*


WEIGHT losses from a gold surface orbiting at 200 km are found by McKeown et al.,[1] the angle between the surface and the direction of motion being 30°. The authors ascribe these weight losses to sputtering of gold by $N_2$, the impact energy being 9 ev, and calculate an erosion rate of 0.1 Å/day or about $5 \times 10^8$ Au atoms/sec-cm². In view of results of sputtering-yield studies at very low bombarding ion energies,[2] it seems doubtful that any detectable sputtering would occur at 9 ev. It seems more likely that the weight losses observed are caused by outgassing. For $N_2$ outgassing, the observed weight losses would correspond to an outgassing rate of $10^{-10}$ torr-liters/sec-cm², which is very low for an unbaked surface.

Aside from this, since the authors indicate that they consider their results to be an upper limit on sputtering from a satellite surface, it is well to point out an apparent arithmetical error. The sputtering yield may be calculated from

$$Y = n/Nv \sin\theta$$

where

$n = 5 \times 10^8$ Au atoms/sec-cm²
$N = 7.82 \times 10^9\ N_2/\text{cm}^3$ (Ref. 3)
$v$ = impact velocity $= 8 \times 10^5$ cm/sec
$\theta = 30°$

These data give $Y = 1.6 \times 10^{-7}\ \text{Au}/N_2$, essentially an order of magnitude lower than the value given by McKeown et al.

### References


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[2] Stuart, R. V. and Wehner, G. K., "Sputtering yields at very low bombarding ion energies," J. Appl. Phys. 33, 2345–2352 (1962).

[3] *U. S. Standard Atmosphere* (U. S. Government Printing Office, Washington, D. C., 1962), p. 81. We follow the assumption by McKeown et al. that the composition in the upper atmosphere is mainly $N_2$.


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* Principal Scientist, Applied Science Division.

## Reply by Authors to R. V. Stuart


D. MCKEOWN* AND M. G. FOX†

*General Dynamics/Astronautics, San Diego, Calif.*

J. J. SCHMIDT‡

*Air Force Cambridge Research Laboratories, Sunnyvale, Calif.*

AND

D. HOPPER§

*Lockheed Missiles and Space Company, Sunnyvale, Calif.*


STUART'S comment on outgassing has been previously considered in the referenced papers[1,2] in which it was concluded that the measured weight loss was due to sputtering. The weight loss was attributed to sputtering and not to out-gassing for the following reasons. First, in making an erosion measurement, not one but four gold surfaces were exposed to the vacuum of space. The gold surfaces were plated on two matched quartz oscillator crystals. Only one of the plated surfaces was actually under molecular bombardment. A photograph of the erosion gage is shown in one of the referenced articles.[2] The output of the gage is the beat frequency of the two crystals. Any outgassing should produce equal mass losses from all four of the gold surfaces, and the frequency of both crystals will increase. This frequency increase is cancelled at the gage output since only the beat frequency is telemetered. The mass change sensed by the gage will be that produced by particles impacting on the surface exposed to the molecular stream.

To check this assumption, a control gage with gold-plated crystals as well as a test gage with gold-plated crystals were flown on Discoverer 26.[1] Mass measurements were not taken until the satellite was in orbit for four days to permit the crystals to outgas. The output of the two gages was monitored closely for the following two days. The results of the test were referenced.[2] The maximum error in measuring the thickness of surface eroded that could be attributed to variation in the power supply voltage, temperature changes, outgassing, and any other unknown cause was found to be ±0.05 Å/day. The erosion rate reported in Ref. 4 was given then as 0.1 ± 0.05 Å/day, and it was reasonable to assume it was caused by sputtering from the results of the previous work.

In calculation of the sputtering yield an apparent error is present if one considers that a satellite has a circular orbit. Satellites do not. Since the atmospheric density drops off rapidly with altitude (between 200 and 330 km, the atmospheric density decreases by an order of magnitude), an average density must be used in calculating the yield. As a result of this density variation, a satellite generates a molecular beam that is intensity modulated as it passes between perigee and apogee.

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* Senior Staff Scientist, Space Science Laboratory. Associate Fellow Member AIAA.

† Development Engineer, Space Science Laboratory.

‡ Staff Scientist, Resident Scientist Office.

§ Flight Test Analytical Engineer.